Jean SALENÇON

Date of birth: 13 November 1940

Member of the Académie des sciences, Paris, France
Member of the Académie des technologies (Emeritus), Paris, France
Member of Academia Europaea
Member of the Istituto Lombardo, Milan, Italy
Foreign Member of the Istituto Lombardo (Milan, Italy)
Foreign Member of the Academia das Ciências de Lisboa (Portugal)
Honorary Member of the Hungarian Academy of Science (Hungary)
Senior Fellow, Hong Kong Institute for Advanced Study, City University, Hong Kong
Honorary Member of the EUROMECH Society
Professor Emeritus, École polytechnique & École nationale des ponts et chaussées, France
Ingénieur général des ponts et chaussées honoraire

RESEARCH ACTIVITIES

Limit Loads Theory under Plane Strain Conditions

I did my first research work at the *Laboratoire de Mécanique de l'École Polytechnique* in April 1964 as a memoir for my civil engineering degree at the *École nationale des ponts et chaussées*. It consisted in the computation of the theoretical bearing capacity of a shallow foundation, based upon the theory of plasticity under plane strain conditions and using the classical method of characteristics as in Sokolovsky's books. It resulted in a short paper in the *Annales des ponts et chaussées* in 1965 [1]. After this first experience, I was accepted as a researcher in this laboratory where, starting from October 1964, under the supervision of J. MANDEL and D. RADENKOVIC, I was assigned various problems to analyse and solve using classical methods within the elastoplastic and viscoelastic frameworks. From what appeared to me as shortcomings in these analyses I decided what was to be the subject of the memoir for my DSc. in Mathematics in 1969: *The Theory of Limit Loads Applied to the Solution of Plane Strain Plasticity Problems*. In this memoir, I underscored the role played by the theory of limit loads both as a guiding principle to design and build solutions to problems of plane-strain free plastic flow and as a variational principle for choosing between different such solutions. It also provided solutions to various punching or indentation problems, insisting on the importance of static and complete solutions. Some of these solutions were subsequently used for applications to metal forming, others in the analysis of bearing capacity of shallow footings on a soil foundation of limited thickness, as discussed later on.

Plasticity and Soil Mechanics

In 1969 at the *École nationale des ponts et chaussées*, I was given the opportunity of delivering a new optional course in Soil Mechanics that was dedicated to fundamental aspects of the discipline derived from the theory of Plasticity. Based upon my personal theoretical research and the practice I had gained in solving problems of bearing capacity of shallow foundations, stability of cavities, etc, I drafted an original course on Plasticity in Soil Mechanics providing the reader with the knowledge of traditional Plasticity theory necessary for the understanding and the implementation of the "plastic" rationale in

[1] Incidentally, the results were confirmed by LYAMIN et *al.* (2007), using a finite element analysis (*Géotechnique,* **57**, 8, 647-662).

Soil Mechanics. As a follow-up, the book *Théorie de la plasticité pour les applications à la mécanique des sols* was published in 1974 by Eyrolles and a revised English version appeared in 1977 with the title *Application of the Theory of Plasticity in Soil Mechanics* (Wiley).

Numerical Solutions

Theoretical calculations of the bearing capacity of shallow foundations were most often carried out applying the theory of plane limit equilibrium in the case of strip footings or the theory of limit equilibrium with axial symmetry under the Haar-Karman hypothesis in the case of circular footings. The problem is thus reduced to the solution of a quasi-linear hyperbolic system of two equations of the 1st order to two variables and two unknown functions, to which the characteristic lines method can be applied. I wrote the first computer codes available in the laboratory for doing these calculations for various types of materials and boundary conditions. These programs were then used by my students and co-workers.

In 1973, I proposed to implement the finite element method in the kinematic method of the theory of limit loads for the solution of plane strain problems in the case of a Tresca material. In a paper with M. FRÉMOND, we highlighted the importance of introducing discontinuities in the velocity fields generated this way and we gave the first results obtained through this approach. Research in that field to develop and generalize this method did not result in an efficient kinematic approach compared with the construction of plane strain or axially symmetrical velocity fields using the characteristic lines method.

The Yield Design Theory

Limit Analysis and the theory of Limit Loads are traditionally dependent in their presentations on strong assumptions on the behaviour of the constituent materials – namely: elasticity, perfect plasticity, principle of maximum plastic work – and even introduce a so-called "rigid-plastic" behaviour which sometimes leads to ambiguous proofs. I realized that the reasoning employed called only upon relatively simple mathematical concepts of convexity and duality, once the resistance of the constituent material had been defined, and did not need to refer to a complete statement of its behaviour in the form of a constitutive equation. In 1976, in a D.E.A. course at the *École nationale des ponts et chaussées*, I introduced the *Theory of Yield Design*, which differs from the theory of limit loads in the fact that it determines the potentialities of resistance of a system in a given geometry, by simply writing that equilibrium of the system and resistance of its constituent material must be mathematically compatible. This is the *primal* or *static interior approach* of the theory of Yield Design. Dualizing this primal approach through the principle of virtual work, which does not call upon any complementary assumption regarding the behaviour of the constituent material, the *kinematic exterior approach* of the theory is obtained using virtual velocity fields as test-functions[2]. The concept of relevant virtual velocity fields, those which lead to non trivial results in the kinematic exterior approach, is purely mathematical. A complete statement of the theory and examples of applications were given in 1983 in the book *Calcul à la rupture et Analyse limite* [Yield Design and Limit Analysis]. A bilingual tutorial (French-English) devoted to this theory was released in 2002 with the book *de l'Élasto-plasticité au Calcul à la rupture* [from Elastoplasticity to Yield Design] (Éditions de l'École polytechnique).
Besides the famous historical reasoning by GALILEO, COULOMB, and MÉRY, the theory of Yield Design proves to cover a very wide range of problems and analyses, especially in civil and construction

[2] This point of view may have been inspired by the second memoir of my DSc. thesis on Linear and non-linear programming and my subsequent paper on the application of linear programming to the yield design of structures, following the works by CERADINI and GAVARINI.

engineering such as methods of plastic analysis of structures or the yield line method for plates and thin slabs and comes out as a theoretical basis of some approaches on the behaviour of reinforced concrete, etc. It is perfectly suited to stability analyses in Soil Mechanics, highlighting the identity of nature existing between the so-called “collapse analyses” and the limit equilibrium methods. It explains the origin of the difficulties encountered in the formulation of some traditional methods for the stability analysis of earth structures such as the circle-line method, which geotechnical engineers managed to overcome from practice by the introduction of complementary assumptions and a statistical calibration of the results so obtained.

On the basis of the Yield Design theory, new and effective methods of analysis can actually be built, which comply with mechanical consistency and get rid of the afore mentioned difficulties, to deal with “classical” structures such as foundations, slopes, embankments…, including the case of anisotropic soils, and with structures for which new design methods have been developed such as tunnels, underwater slopes, reinforced soil structures. For this latter type of structures, it sometimes proves worth combining the theory of Yield Design with a homogenization process.

As a follow-up to the course I delivered during three years on that topic at the City University of Hong Kong, I published a book that provides a general presentation of the theory together with applications to continuous media, one-dimension curvilinear media, plates and thin slabs, probabilistic approach and dimensioning, and a rigorous framework for Ultimate Limit State design (*Yield Design*, Wiley, 2013; *Yield Design teaching material HK*, 2013).

The development of soil reinforcement techniques was at the origin of new design codes for earth made structures. When taking part in the drafting of the French *CLOUTERRE* recommendations, I realized that the implementation of the concept of Ultimate Limit State Design (ULSD) was confronted with an essential difficulty in taking into account the resistance of the reinforcing elements. In fact, it requires that a clear distinction be made between the active forces, whose design values are assigned, and the resistances which are only defined by their design limit values independently of the active forces. (This distinction had been clearly stated in Coulomb’s famous Essay). The theory of Yield Design obviously meets that goal and stands as the fundamental theoretical basis of ULSD. Through its kinematic approach, it provides a precise and complete formulation of the fundamental inequality of ULSD where the “effect of the active forces” is their rate of work and the “effect of the resistances” is the maximum resisting rate of work, the inequality being written for any virtual kinematically admissible velocity field. It explains why a “method- or model coefficient” shall be introduced for practical implementation of the method when considering only a limited number of virtual velocity fields.

As a concrete and practical application of the theory, we devised the *STARS* software for stability analysis of reinforced soil slopes and walls, which is at the same time remarkably fast and specifically suited to ULSD (A. ANTHOINE, P.DE BUHAN, L. DORMIEUX & J. SALENÇON, 1990-1991). In its most recent version, the *TALREN 4* commercial software, produced by *TERRASOL,* explicitly refers to and makes use of the Yield Design approach (cf. *Ground Engineering,* September 2009, 20-25).

Optimality and Probability Approaches of Yield Design

In the Yield Design theory, the loads applied to a system and the resistances of its constituent materials play symmetric roles in the equations to be satisfied for potential stability. The systems under concern in our analyses are composed of a finite number *m* of zones where the domain of resistance of the constituent material depends linearly on one scalar resistance parameter. This corresponding set of *m* resistance parameters defines the design- or dimensioning vector of the system. Given a set of prescribed loads, potential stability of the system defines potentially safe designs. Optimal dimensioning requires minimising a given objective function on the convex domain of potentially safe designs and leads to linear or convex programming problems.

The introduction of a probabilistic viewpoint was, in particular, the subject of the Doctor-Engineer thesis of A. CARMASOL under my supervision. When the prescribed loads and the assumed resistances are given a stochastic character, the question of potential stability can only receive a probabilistic answer. The interior approach and, essentially, the kinematic exterior approach provide lower- and upper bound estimates for the probability of stability and the probability of collapse. The relevant concept is shown to be that of "domain of potential stability" generated, in the product linear space of the loading and resistance parameters, by the loads and design parameters for which the probability of stability of the system is at least equal to a prescribed level. Interior and exterior approaches of this domain can be obtained. Accuracy of the proposed methods was proven and the importance of the choice of the probability law assigned to the design vector was enhanced, especially as regards the distribution of the extreme values of the design parameters since high levels of reliability are usually required. Also, comparison with already existing approaches has shown that they might have exhibit a risk of underestimating low probabilities of collapse.

Homogenization and Yield Design

After having been used for centuries, or even millennia, soil reinforcement techniques were considerably developed with the appearance of REINFORCED EARTH®, Soil nailing, Geotextiles, Lime columns, TEXSOL®, etc. Stability analyses of earth structures made with such materials were performed adapting classical methods that existed for natural soils. The implementation of the Yield Design theory for such analyses was straightforward as already stated in a preceding paragraph, but it turned out that an alternative approach could be obtained through the homogenization method that had been extensively developed for the analysis of composite materials within the framework of classical constitutive equations (Elasticity, Plasticity…).
After the pioneering work by P. SUQUET within the framework of Elastoplasticity, P. de BUHAN implemented the homogenization method in the Yield Design theory. This was the subject of his DSc. thesis under my supervision. He showed that, if a high scale ratio between the considered earth structure and the reinforcement is maintained and if the reinforcement is periodical, it is possible to define a homogeneous material associated with the reinforced soil in such a way that stability analyses performed on a geometrically identical structure made of this homogeneous material be significant for the actual structure. Those two conditions happen to be quite often met in practical cases.

In the case of a purely cohesive soil, reinforced by a purely cohesive material (*e.g.,* lime columns) the associated homogeneous material is completely determined in the form of a purely cohesive and anisotropic material. Examples such as the stability analysis of slopes or the bearing capacity of strip footings have shown that the "homogenized" analysis is, at the same time, more convenient when applying the static approach and more efficient and relevant in the kinematic one when the same class of virtual collapse mechanisms is used, leading to a stability factor lower by some 20 %. A simple explanation of this important result lies in the fact that the scale change leading from the reinforced soil to the homogeneous material involves an auxiliary Yield Design problem, which means that, when applied to the homogenized structure, any given virtual collapse mechanism is in fact "thinner" than when applied directly to the original one. One may also say that any direct Yield Design calculation on the original structure implicitly results in considering the reinforced original material as a homogeneous isotropic one that is stronger than the anisotropic homogeneous equivalent material.

When the soil is reinforced by means of inclusions, the same type of analysis made it possible, for instance, to determine the strength criterion of the homogeneous anisotropic material equivalent to REINFORCED EARTH® and to perform stability analyses (DSc. thesis of L. SIAD). Comparisons with classical methods have shown that the improvement in the estimate of the stability factor is highly depending on the considered structure (from a few per cents to several tens), which provides a way of checking the quality of the structure design in order to improve the arrangement of the reinforcing elements.

With the experience of soil mechanics problems, we applied the same rationale to the determination of the yield strength of long fibre composite materials.

Yield Design Analyses for Anisotropic Cohesive Soils

Due to the consolidation process, anisotropy is often encountered when dealing with purely cohesive soils. A review of the methods used for the stability analysis of earth structures made of such soils shows that, being derived from the methods that are relevant for isotropic soils, they come against important difficulties from the mechanical point of view which may lead to ambiguous or even contradictory results.

In order to apply the Yield Design theory properly it was first necessary to define a three-dimensional yield criterion for an anisotropic soil based upon experimental results available in the literature in the form of the cohesion polar diagram of a transversally isotropic material. The criterion involves two anisotropy ratios. The maximum resisting rate of work functions for this anisotropic criterion were explicitly determined and, from the Yield Design theory, we could devise and implement mechanically consistent methods for the stability analysis of the most frequently studied earth structures (Doctor-Engineer thesis of A. TRISTÁN-LOPEZ):

- *Bearing capacity of strip footings*: assessing the impact of anisotropy, it was shown that, in some cases, a bearing capacity calculation within the isotropic framework on the basis of triaxial tests performed on vertical samples may result in a 30 % overestimation (the results were given in the form of charts).
- *Stability analysis of slopes and fills*: with the knowledge of the maximum resisting rate of work functions for the anisotropic criterion, it was possible to use the kinematic exterior approach properly, without any mechanical inconsistency. Rotational rigid block virtual collapse mechanisms were used for stability analyses of slopes and fills and the obtained results brought out the influence of each anisotropy ratio on the stability of the considered earth structure. Practical relevance was shown by applying the method to actual collapses recorded in the literature using the corresponding available data and parameters. It turned out that the results obtained were in complete agreement with the reported collapse features.

Yield Design in the Case of Seismic Loading

As reported previously, the *STARS* software includes the possibility of analysing the stability of reinforced soil slopes taking into account seismic forces through a quasi-static approach. In a long-lasting collaborative work with A. PECKER, we considered the stability analysis of surface footings in the same conditions. The quasi-static approach results in the determination of the bearing capacity of the footing under an inclined and eccentric load with bulk forces in the foundation soil consisting of gravity forces and seismic acceleration.

The practical problems to be dealt with were concerned with the case of a strip footing on a purely cohesive soil. The bearing capacity of the footing was given in the form of three-dimensional charts for the three loading parameters – vertical load, horizontal load, eccentricity or tilting moment – first in the classical case of a purely cohesive soil (Tresca's criterion) and then for a purely cohesive soil with zero-tension cut-off. The same problem was studied afterwards by L. VERZURA adding cohesion anisotropy according to Bishop's law. The results were first applied to the practical analysis of the stability of buildings in Mexico City following the 1985 Michoacán earthquake (G. AUVINET, A. PECKER, M. ROMO, L. VERZURA & J. SALENÇON). Later on, the results were extended to the case of rigid inclusion reinforcement and were applied to the design of the foundations for the Rion-Antirion viaduct (Charilaos Trikoupis Bridge) in Greece by A. PECKER. As a follow up, the bearing capacity of circular footings under seismic loading was the theme of the DSc. thesis of C. T. CHATZIGOGOS supervised by A. PECKER and myself. It is worth noting that, in June 2008, the Charilaos Trikoupis viaduct was subjected to a strong earthquake with moment magnitude $M_w = 6.5$ and behaved as anticipated without structural damage. This innovative design concept for a foundation has been replicated at least for two major suspension bridges in Turkey and one nuclear waste storage in France.

In addition to the quasi-static approach, the concept of performance-based design has been introduced for the design of seismically loaded structures. It requires having a model of the whole structure, namely the footing itself, the interface and the foundation soil, in order to be able to calculate its response all along the earthquake and to anticipate the final values of the relevant parameters at the end of the seism. For this purpose, we developed a dynamic macroelement model which takes into consideration, independently from each other, the non-linear mechanisms involved in the global response of the system: sliding at the interface, foundation soil yielding in the vicinity of the interface and uplift detachment of the footing at the interface.

Yield Design and Bearing Capacity Problems

In my fist publications devoted to the calculation of the theoretical bearing capacity of surface footings (shallow foundations), I adopted the classical framework of the superposition method and, applying the theory of limit equilibrium together with the method of characteristics, I devoted myself to the calculation of the bearing capacity factors of strip footings (plane limit equilibrium) and circular footings (with the Haar-Karman hypothesis), either when no solution was available, or when estimates of the bearing capacity factors could be improved (*e.g.* strip footing on a soil layer of limited thickness with special attention to the role of friction between the soil layer and the bedrock, purely cohesive or frictional cohesive soil with variable cohesion).

Having written the first computer codes to solve these problems, I decided to depart from the superposition method and to compute bearing capacities as a whole, thus taking into account the coupling effects between soil cohesion, surface load and gravity forces. In order to make the results practically available, they were given in the form of multi-entry charts providing the two correction factors to be applied to the popular superposition method: the coupling effect factor and the shape factor (between a strip footing and a circular one). Such a presentation was made possible thanks to the use of dimensional analysis and to a theorem I had established proving the equivalence between a vertical cohesion gradient and gravity forces in the case of Coulomb's criterion.

Other aspects of my research on the bearing capacity of shallow foundations were already mentioned in preceding paragraphs (anisotropy, reinforced soils, seismic forces, confining effect, etc.). It may be worth noting that the presentation of the bearing capacity of a strip footing in the form of 3-dimensional charts for the three loading parameters – vertical load, horizontal load, eccentricity or tilting moment – was some kind of a "première", which has been adopted by other authors afterwards.

This long-lasting research project was carried out in collaboration with D. BERTHET, J.-C. HAYOT, M. BARBIER, M. BEAUBAT, M. CROC, G. MICHEL, A. PECKER, C. ROCHE, P. FLORENTIN, Y. GABRIEL, M. MATAR, F. MAZUEL, J.-P. MICHEL, A. TRISTÁN-LOPEZ, Z. KHOSRAVI, P. de BUHAN, L. SIAD, A. ANTHOINE, C. T. CHATZIGOGOS (in chronological order).

Textbooks

Related to my teaching activities in French engineering schools, I wrote several textbooks in French, devoted to Continuum Mechanics, Elastoplasticity and Viscoelasticity, which were periodically revised and republished. An English version of the last-but-one edition of my textbook on Continuum Mechanics, translated by Stephen, LYLE, appeared in 2001 (*Handbook of Continuum Mechanics*, Springer).

Appointed as a Senior Fellow of the Hong Kong Institute for Advanced Study (HKIAS) City University, in 2016, I committed myself to writing a consistent set of a few textbooks in English on the subjects mentioned above, where I could take advantage of the 10-year hindsight from my last teaching position in France and the experience I had gained when teaching at City-U. Within this new framework, I could revisit these topics adding an historical viewpoint, checking original references and introducing some original personal results: *Viscoelastic Modeling for Structural Analysis* (Wiley, 2019) and *Elastoplastic Modeling* (Wiley, 2020). On this latter occasion, I came to investigate Tresca's original memoirs, considered by KOITER "as the birth of the mathematical theory of plasticity", and organize a presentation of this classical theory in the form of a follow-up to Tresca's legacy.

As another example, I would mention my historical investigation about the concepts of virtual velocities and virtual rate of work. This modest work in comparison with the huge contributions by P. DUHEM and R. DUGAS was the topic of some lectures I gave, among which my inaugural lecture at the Hungarian Academy of Sciences. It turned out also to be the cornerstone of the course on Continuum Mechanics I delivered at the City University of Hong Kong in 2014 & 2016 in a form deliberately focused on the concept of modelling, underscoring its basic geometrical assumptions and the "rules of the game", which include validation as a primary requirement. For the PhD students who were attending the course, this robust approach appeared as lighting the way back to their previous knowledge on the topic. The book *Virtual Work Approach to Mechanical Modeling* (Wiley, 2018) was written as a follow-up to this course (*Continuum Mechanics teaching material HK*, 2014). In the same "historical" spirit, taking advantage of two significant celebrations – namely, the 250th anniversary of Coulomb's *Essai* (1773) and 150th anniversary of Tresca's Memoirs on the Fluidity of solids (1864-1870) – I published three papers, devoted to Coulomb's and Tresca's legacies in Soil mechanics and the mathematical theory of Plasticity respectively, and triggered the organization of the Colloquium "*Charles Augustin Coulomb: un hommage géotechnique*" by the French Society for Soil Mechanics and Geotechnics.

Thematic Publication Index

Major Topics	Publication numbers
Anisotropy	78, 79, 82, 83, 85, 87, 97, 101, 105, 206
Cavities	2, 11, 12, 27, 126
Characteristic lines	1, 37, 43, 53, 54, 216
Composites materials	108, 112, 144
Continuum mechanics	106, 138, 154, 157, 165, 170, 173, 164, 169, 174, 192, 194, 195, 196, 197, 198, 202, 204, 205, 208, 209, 213, 224, 228, 230, 232, 233
Dynamic actions	134, 135
Eccentric loads	131, 132,149
Elastoplasticity	26, 59, 99, 160, 200, 216, 219, 222
Energy	186, 187, 189
Finite element method	30, 40, 52
Heterogeneity	19, 29, 35, 36, 48, 51, 71, 76, 77, 172
Homogenization	101, 103, 104, 144
Inclined loads	131, 132, 149
Limit analysis	20, 26, 32, 37, 39, 53, 90, 161. 190’, 194, 216
Limit loads	1, 17, 23, 24, 25, 37, 53, 128, 161, 190’, 194, 216
Miscellanea	102, 199, 201, 206, 210, 226, 227, 239
Non-standard materials	23, 24, 25, 37, 53, 90
Optimization	13’, 16, 42, 107, 190’, 194
Plasticity in soil mechanics (theory)	37, 39, 53
Probabilistic approach	13’’, 90, 91, 96, 190’, 194, 212
Punch indentation	5, 6, 7, 8, 9, 19
Reinforced soils	95, 101, 105, 107, 113, 119, 123, 126, 144, 148, 153
Seismic actions	113, 116, 134, 135, 141, 145, 146, 153, 166, 168, 171, 172, 179, 180, 184, 188, 203
Stability of slopes and fills	55, 65, 67, 79, 82, 113, 118, 123, 148, 240
Surface footings (circular)	31, 47, 75, 76, 77, 141, 145, 159, 166, 168, 171, 172, 178, 188
Surface footings (strip)	21, 29, 71, 51, 56, 68, 83, 116, 131, 132, 141, 145, 149, 158, 162, 171, 178, 179, 184
Two-layer	7, 9, 21, 31, 47, 51, 71, 75, 76
Ultimate limit states	123, 125, 129,
Variable cohesion	19, 35, 36, 51, 71, 75, 76
Viscoelasticity	89, 177, 207
Yield design	16, 55, 65, 86, 90, 109, 110, 118, 124, 128, 160, 161, 190’, 194, 211, 212, 216, 223, 225, 234, 235, 236
Yield Design: Static extensions	13, 33, 37, 53, 131