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K. Chouayakh, M. Rachik, K. Satori, C. El Bekkali & I. Elmouki - *A grazer-predator model with optimal fishing efforts.*

M. Bentounsi, I. Agmour, N. Achtaich & Y. El Foutayeni - *The influence of price on the profit of fishermen exploiting prey, middle- predator and top-predator fish populations.*

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A GRAZER-PREDATOR MODEL WITH OPTIMAL FISHING EFFORT

ABSTRACT

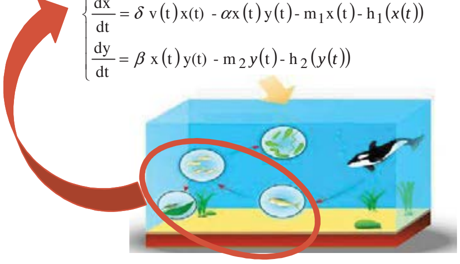
In this talk, we present a mathematical model in the form of three differential equations which describe dynamics of aquatic plants, grazers and predators fishes populations in the absence and presence of fishing fleets. We introduce two controls variables to discuss the impact of optimal fishing effort on the environmental sustainability and bioeconomy.

Keywords: Aquatic plants, Fish harvesting, Grazer-predator fishes, Optimal control, Environmental sustainability, Bioeconomy.

THE GRAZER-PREDATOR MODEL

The hydraulic food chain differential system with harvesting is modeled using the following ordinary equations:

$$\begin{cases} \dfrac{dv}{dt} = r\,v(t)\left(1 - \dfrac{v(t)}{K}\right) - \gamma x(t)\, v(t) - mv(t) \\ \dfrac{dx}{dt} = \delta\, v(t)x(t) - \alpha x(t)y(t) - m_1 x(t) - h_1(x(t)) \\ \dfrac{dy}{dt} = \beta\, x(t)y(t) - m_2 y(t) - h_2(y(t)) \end{cases}$$



with $v(0)=v_0$, $x(0)=x_0$ and $y(0)=y_0$ as given initial conditions and where

- r>0 is the intrinsic growth rate of the vegetation in the absence of the grazers,
- k>0 is the vegetation carrying capacity,
- m is nongrazing mortality of vegetation,
- m_1 and m_2 are the natural mortality rates of vegetation, grazers and predators respectively,
- α, β, γ and δ represent the efficiency by which a marine fish specimen is converted to an other one,
- functions $h_1(x(t))$, $h_2(y(t))$ are non-negative and represent the harvesting of grazers and predators respectively;

we note that $h_1(x(t))= q_1 e_1(t)\,(x(t))$ and $h_2(y(t))=q_2 e_2(t)\,(y(t))$ with q_1, q_2 are the catchability coefficients, and $e_1(t)$, $e_2(t)$ are the fishing effort associated to x and y state variables respectively.

OPTIMAL FISHING EFFORT

THE ENVIRONMENTAL SUSTAINABILITY CASE

Our main goal is to suggest an optimal harvesting policy, which concerns the maximization of the harvesting functions $h_1(x(t))$ and $h_2(y(t))$ while minimizing the fishing effort functions $e_1(t)$ and $e_2(t)$, related to the following objective function *J* :

$$\max\ J(e_1,e_2) = \int_0^T \left[a\, h_1(t) + b\, h_2(t) - A\, e_1^2(t) - B\, e_2^2(t)\right] dt$$

We seek two optimal control functions e_1* and e_2* satisfying:

$$\max_{(e_1,e_2)\in E^2} J(e_1,e_2) = J(e_1^*,e_2^*)$$

$$\text{with } E = \left\{(e_1,e_2) \,\middle|\, 0 \le e_1(t), e_2(t) \le 1,\ e_1, e_1 \text{ measurable},\ t \in [0,T]\right\}$$

the set of admissible control.

Theorem (Necessary conditions and Characterization)
Given two optimal controls e_1* and e_2*, along with solutions *v**, *x** and *y** of the corresponding state system, there exist adjoint variables λ_1, λ_2 and λ_3 satisfying :

$$\lambda_1' = \lambda_1\left[r\left(\frac{2}{K}v(t) - 1\right) + \gamma\, x(t) + m\right] - \lambda_2\, \delta\, x(t)$$

$$\lambda_2' = -aq_1e_1(t) + \lambda_1\, \gamma\, v(t) - \lambda_2\left[\delta\, v(t) - \alpha\, y(t) - m_1 - q_1e_1(t)\right] - \lambda_3\, \beta\, y(t)$$

$$\lambda_3' = -bq_2e_2(t) + \lambda_2\, \alpha\, x(t) - \lambda_3\left[\beta\, x(t) - m_2 - q_2e_2(t)\right]$$

with $\lambda_1(T)=\lambda_2(T)=\lambda_3(T)=0$ are the transversality conditions.

Optimal controls will be characterized by:

$$e_1^*(t) = \min\left(\max\left(0, \frac{q_1 x(t)}{2A}(a-\lambda_2), 1\right)\right),\quad e_2^*(t) = \min\left(\max\left(0, \frac{q_2 y(t)}{2B}(b-\lambda_3), 1\right)\right).$$

THE BIOECONOMIC CASE

The incorporation of economic considerations into resource harvesting models leads to the subject called bioeconomics.
we introduce the two same control functions e_1 and e_2 but associated in this case, to profits functions $\pi_1(e_1)$ and $\pi_2(e_2)$ respectively.

$$\text{profit } (\pi_i) = \text{Total revenue } (R_i) - \text{total cost } (C_i)$$

where

$$R_i = p_i h_i \text{ with } p_i: \text{price and } h_i: \text{harvesting}.$$

and

$$C_i = c_i e_i \text{ with } c_i: \textit{coast} \text{ and } e_i: \text{fishing effort}.$$

For this, we propose an optimization criterion subject to system which is defined by the following objective function *J*:

$$\max\ J(e_1,e_2) = \int_0^\infty e^{-\theta t}\left[\Pi_1(e_1) + \Pi_2(e_2)\right] dt$$

where θ is the annual discount rate.

Explicity, *J* is defined as

$$J(e_1,e_2) = \int_0^\infty e^{-\theta t}\left[p_1q_1\, e_1(t)x(t) + p_2q_2\, e_2(t)y(t) - c_1\, e_1(t) - c_2\, e_2(t)\right] dt$$

Therefore, the main goal, concerns the characterization of the two sought optimal controls e_1* and e_2* such that

$$\max_{0\le e_1,e_2\le 1} J(e_1,e_2) = J(e_1^*,e_2^*)$$

Similarly, we can easily prove the existence of an optimal control pair $e = (e_1, e_2)$ satisfying the condition of maximum. In the following, we announce the theorem of necessary conditions and characterization associated to the bioeconomic case.

Theorem
Given two optimal controls e_1* and e_2* and solutions *v**, *x** and *y** of the corresponding state system, there exist adjoint variables λ_1, λ_2 and λ_3 defined by

$$\begin{cases} \dfrac{d\lambda_1}{dt} = -\lambda_1\left(r\left(1 - \dfrac{2v(t)}{K}\right) - \gamma x(t) - m\right) - \lambda_2 \delta x(t) \\ \dfrac{d\lambda_2}{dt} = -e^{-\theta t} p_1q_1e_1(t) + \lambda_1\gamma\, v(t) - \lambda_2\left(\delta v(t) - \alpha y(t) - m_1 - q_1e_1(t)\right) - \lambda_3\beta y(t) \\ \dfrac{d\lambda_3}{dt} = -e^{-\theta t} p_2q_2e_2(t) + \lambda_2\alpha\, x(t) - \lambda_3\left(\beta x(t) - m_2 - q_2e_2(t)\right) \end{cases}$$

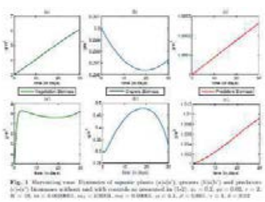
with the transversality conditions $\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = 0.$
which imply that e_1* and e_2* are defined by the following analytical formulations

$$e_1^*(t) = \left(\frac{p_1x^*(t)}{c_1} - \frac{1}{q_1}\right)\left(\theta + \frac{\lambda_1(t)\gamma v^*(t)}{e^{-\theta t}\left(p_1 - \dfrac{c_1}{q_1x^*(t)}\right)} - \delta v^*(t) + \alpha y^*(t) + m_1\right) - \frac{\beta y^*(t)x^*(t)}{c_1}\left(p_2 - \frac{c_2}{q_2y^*(t)}\right)$$

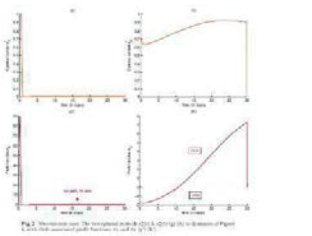
$$e_2^*(t) = (\theta - \beta x^*(t) + m_2)\left(\frac{p_2y^*(t)}{c_2} - \frac{1}{q_2}\right) + \frac{\alpha y^*(t)x^*(t)}{c_2}\left(p_1 - \frac{c_1}{q_1x^*(t)}\right)$$

NUMERICALS SIMULATIONS

1st case



2nd case



CONCLUSION

In this talk, we devised a mathematical model which describes dynamics of a food chain composed by three different marine species; the aquatic plants, grazers and predators. We suggested two optimal harvesting policies. Based on the optimal control theory, we considered a first optimal harvesting policy for the environmental sustainability case, and which has aimed to minimize two fishing efforts functions related to grazer and predator states variables respectively for not affecting the trophic-halieutic environment, while finding the possibility to maximize their associated harvesting functions during fishing fleets, for the benefice of fishermen. As regards to the second optimal harvesting policy proposed for the bioeconomic case, it has aimed to minimize the two same fishing efforts functions but focusing only on the maximization of their associated profits functions, as an example of the effectiveness of the optimal harvesting strategy in the bioeconomy case.

The influence of price on the profit of fishermen exploiting prey, middle-predator and top-predator fish populations

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Introduction

An interesting and attractive topic in population ecology is understanding the diversity and community composition of populations that ultimately determine the overall stability of an ecosystem. In this work, we consider a tri-trophic prey-predator model which consists of three constituent populations; i.e., prey, middle predator and top predator. The objective of this work is to show how changes in the price can affect the profits of fishermen which exploit these three marine species. To achieve this aim we define a bioeconomic equilibrium model of this three populations then we compute the generalized Nash equilibrium point.

Problematic

Bioeconomic model

Let x_1, x_2 and x_3 respectively denoted the prey, middle predator and top predator populations sizes. The resulting system of equations reads as follows:

$$\begin{cases} \frac{dx_1}{dt} = r_1x_1(1-x_1) - \alpha x_1x_2 - \beta x_1x_3 - C_1 \\ \frac{dx_2}{dt} = r_2y(1-y) + \bar{\alpha} x_1x_2 - \delta x_2x_3 - C_2 \\ \frac{dx_3}{dt} = r_3x_3(1-x_3) + \bar{\beta} x_1x_3 + \bar{\delta} x_2x_3 - C_3 \end{cases}$$

r_1 : The intrinsic growth rate of the prey.
r_2 : The intrinsic growth rate of the middle-predator.
r_3 : The intrinsic growth rate of the top-predator.
α: Capture rate of the prey by predator.
β: Capture rate of the prey by top-predator.
$\bar{\alpha}$: The middle-predators' assimilation rates for competition with prey.
δ: Mortality rate of middle predator by top-predator.
$\bar{\beta}$: The top predators' assimilation rates for compitition with prey.
$\bar{\delta}$: The top predators' assimilation rates for compition with middle-predator.
$C_i = q_iE_ix_i$: The total catches of fish population i.
E_i: The fishing efforts to exploit a species j.
q_i : The catchability coefficients of species j.

The problem of determining the fishing effort that maximizes the profit of each fisherman leads to a Nash equilibrium problem.

Nash Equilibrium Problem (NEP)

The first fisherman must solve the problem (P1)

$$\begin{cases} \max \pi_1(E^1) = < E^1, -P_0qAE^1 + P_0qX^* - c^1 - P_0qAE^2 > \\ sc: \quad AE^1 \leq X^* - AE^2 \\ \quad E^1 \geq 0, (E^2) \ given \end{cases}$$

The second fisherman must solve the problem (P2)

$$\begin{cases} \max \pi_2(E^2) = < E^2, -P_0qAE^2 + P_0qX^* - c^2 - P_0qAE^1 > \\ sc: \quad AE^2 \leq X^* - AE^1 \\ \quad E^2 \geq 0, (E^1) \ given \end{cases}$$

Definition

The point (E^1, E^2) is called Nash equilibrium point if and only if E^1 is a solution of problem (P1) for E^2 given, and E^2 is solution of problem (P2) for E^1 given.

The essential conditions of Karush-Kuhn-Tucker applied to (P1)

$$\begin{cases} 2P_0qAE^1 + c^1 - P_0qX^* + P_0qAE^2 - u^1 + A^T\lambda^1 = 0 \\ AE^1 + v^1 = X^* - AE^2 \\ \langle u^1, E^1 \rangle = \langle \lambda^1, v^1 \rangle = 0 \end{cases}$$

The essential conditions of Karush-Kuhn-Tucker applied to (P2)

$$\begin{cases} 2P_0qAE^2 + c^2 - P_0qX^* + P_0qAE^2 - u^2 + A^T\lambda^2 = 0 \\ AE^2 + v^2 = X^* - AE^1 \\ \langle u^2, E^2 \rangle = \langle \lambda^2, v^2 \rangle = 0 \end{cases}$$

$$\Rightarrow \begin{cases} u^1 = 2P_0qAE^1 + P_0qAE^2 + c^1 - P_0qX^* \\ u^2 = 2P_0qAE^2 + P_0qAE^1 + c^2 - P_0qX^* \\ v = X^* - AE^1 - AE^2 \\ \langle u^i, E^i \rangle = \langle \lambda^i, v^i \rangle = 0 \quad \forall i = 1,2 \\ u^i, E^i, v \geq 0 \quad \forall i = 1,2 \end{cases}$$

Linear Complementarity Problem (LCP)

Find vectors $z, w \in IR^9$ such that:

$$\begin{cases} w = Mz + b \geq 0 \\ z, w \geq 0 \\ z^T w = 0 \end{cases}$$

with $w = \begin{pmatrix} u^1 \\ u^2 \\ 0 \end{pmatrix}$, $M = \begin{bmatrix} 2P_0qA & P_0qA & A^T \\ P_0qA & 2P_0qA & 0 \\ -A & -A & 0 \end{bmatrix}$, $z = \begin{pmatrix} E^1 \\ E^2 \\ 0 \end{pmatrix}$, $b = \begin{pmatrix} c^1 - P_0qX^* \\ c^2 - P_0qX^* \\ X^* \end{pmatrix}$

Results and discussions

Solution of LCP:

The solution of the linear complementarity problem ***LCP(M,b)*** is the vector $z^* = (E_1^*, E_2^*, 0)^T$

where $$\begin{cases} E_1^* = \frac{1}{3}A^{-1}\left(X^* - \frac{c^1}{P_0q}\right) \\ E_2^* = \frac{1}{3}A^{-1}\left(X^* - \frac{c^2}{P_0q}\right) \end{cases}$$

Solution of NEP "Fishing effort":

The solution of the Nash equilibrium problem is the point $(E_1^*, E_2^*)^T$

Now, we take as case of study two fishermen who catch the three fish species. In order to assure the existence and stability of the locally asymptotically stable state of the three fish populations we consider the parameters of the model system as

Prey	Middle-predator	Top-predator
$r_1=0,5$	$r_2=0,3$	$r_3=0,2$
$\alpha=2.10^{-4}$	$\bar{\alpha}=10^{-5}$	$\bar{\beta}=10^{-4}$
$\beta=3.10^{-4}$	$\delta=2.10^{-5}$	$\bar{\delta}=10^{-4}$
$q_1=0,1$	$q_2=0,02$	$q_3=0,004$
$P_1=1$	$P_2=2$	$P_3=3$
$c_1=0,1$	$c_1=0,1$	$c_1=0,1$
$c_2=0,2$	$c_2=0,2$	$c_2=0,2$

Table 1. Characteristics of fish populations.

The influence of the number of the price

P_1	P_2	P_3	E_1	E_2	H_1	H_2	π_1	π_2
1	2	3	17,0451	16,5151	245,0957	234,4651	282	269
16	27	48	17,6383	17,6073	246,5725	246,2429	4513	4500
51	70	108	17,6627	17,6492	246,6552	246,5718	13584	13567
106	133	327	17,6749	17,6702	246,6923	246,6334	29111	29099
808	811	914	17,6794	17,6778	246,7095	246,6775	200558	200531
1000	1079	1090	17,6797	17,6784	246,7107	246,6839	249295	249266

Table 2. The influence of the price on the fishing effort, catches and profits.

According to tables 2, one can remark that an increase in the price leads to an increase in fishing effort, catches and profit. But it is clear that when the price level increases significantly, ie when it varies in a large amplitude interval, the fishing effort and the catches increase by varying in an interval of small amplitude. More precisely, when the price is between 1 and 1090, the fishing effort varies between 16,51 and 17,68, and the catches vary between 234,4 and 246,7. This is justified by the need for conservation of marine species even if the price increases significantly.
One can see that the level of profit increases, which allows fishermen to have highest returns through more reasonable catches, taking into account the conservation of biodiversity.
These results allow us to deduce that our model is pertinent since it allows us to determine the fishing effort that maximizes the profit of each fisherman without being obliged to make more catches that lead to the overexploitation of these marine species.
Let us add that when the price tends to infinity, the fishing efforts of the two fishermen are equal and they do not exceed 18, as well as the catches which do not exceed 250, contrariwise the profit, is always increasing thanks to the increase of the price. Then we can deduce the effect of the price change on the
fishing effort, catches and profit.
It is very interesting to note that if the price tends to infinity and the fishing effort is superior than 18, then the catches and the profit decrease.

Mathematics for economic progress and ecological balance

In this work, we have treated a tri-trophic fishing model, namely the top-predator population that predominates both on the middle-predator populations and the prey. We adopted the approach according to which each fisherman can maximize these benefits to the biological equilibrium, depending on the fishing effort devoted to these marine species. From this study, this problem generates a generalized problem of Nash equilibrium, Which produces a transformation into a problem of linear complementarity in order to solve the problem. A numerical example verifying our theoretical results is also included.