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Posters de la session *Mathématiques appliquées à l’exploitation des ressources marines*
ABSTRACT

In this talk, we present a mathematical model in the form of three differential equations which describe dynamics of aquatic plants, grazers and predators fishes populations in the absence and presence of fishing fleets. We introduce two control variables to discuss the impact of optimal fishing effort on the environmental sustainability and bioeconomy.

Keywords: Aquatic plants, Fish harvesting, Grazer-predator fishes, Optimal control, Environmental sustainability, Bioeconomy.

THE GRASSER-PREDATOR MODEL

The hydraulic food chain differential system with harvesting is modeled using the following ordinary equations:

\[
\begin{align*}
\frac{dx}{dt} &= r(x)(1-\frac{x}{K})-d_1x+y(t)-m_1x(t)-b_1(y(t)) \\
\frac{dy}{dt} &= -d_2y - ax_1(t) - m_2x(t) - b_2(y(t)) \\
\frac{dv}{dt} &= -d_3v - b_3(y(t)) - b_4(y(t)) \\
\end{align*}
\]

with \(y(0)=y_0, x(0)=x_0,\) as given initial conditions and where
- \(r>0\) is the intrinsic growth rate of the vegetation in the absence of the grazers,
- \(K>0\) is the vegetation carrying capacity,
- \(m\) is nongrazing mortality of vegetation,
- \(m_1, m_2\) are the natural mortality rates of vegetation, grazers and predators respectively,
- \(a, b_1, b_2, b_3, b_4\) represent the efficiency by which a marine fish specimen is converted to an environmental sustainability and bioeconomy.

THE BIOECONOMIC CASE

The incorporation of economic considerations into resource harvesting models leads to the subject called bioeconomics. As regards to the second optimal harvesting policy proposed for the bioeconomic case, it we introduce the same two control functions \(e_1\) and \(e_2\) but associated in this case, to profits functions \(\Pi(e_1)\) and \(\Pi(e_2)\) respectively.

\[
\begin{align*}
\text{profit } (e_1) = \text{Total revenue } (K) - \text{total cost } (C_e) \\
\text{where } \quad R_t = p_i \text{ price and } h_i \text{ harvesting}.
\end{align*}
\]

and

\[
\begin{align*}
\text{profit } (e_2) = \text{profit } (e_1) - e_1 - e_2 \\
\text{where } \theta \text{ is the annual discount rate.}
\end{align*}
\]

Explicitly, \(J\) is defined as

\[
J(e_1, e_2) = \int e^{\theta t} \left[p_i(q_1(t))x(t) + p_2 q_2(t)x(t) - c_1 e_1(t) - c_2 e_2(t)\right] dt
\]

Therefore, the main goal, concerns the characterization of the two sought optimal controls \(e_1^*\) and \(e_2^*\) such that

\[
\max_{(e_1, e_2) \in E^2} J(e_1, e_2) = J(e_1^*, e_2^*)
\]

Similarly, we can easily prove the existence of an optimal control pair \(e = (e_1, e_2)\) satisfying the condition of maximum. In the following, we announce the theorem of necessary conditions and characterization associated to the bioeconomic case.

Theorem

Given two optimal controls \(e_1^*\) and \(e_2^*\) along with solutions \(x^*, y^*, z^*\) of the corresponding state system, there exist adjoint variables \(\lambda_1, \lambda_2, \lambda_3\) defined by

\[
\begin{align*}
\frac{d\lambda_1}{dt} &= -\lambda_1 + \lambda_2 + \lambda_3 \\
\frac{d\lambda_2}{dt} &= -\lambda_2 + \lambda_3 \\
\frac{d\lambda_3}{dt} &= -\lambda_3
\end{align*}
\]

with the transversality conditions \(\lambda_1(T) = \lambda_2(T) = \lambda_3(T) = 0\), which imply that \(e_1^*\) and \(e_2^*\) are defined by the following analytical formulations

\[
\begin{align*}
e_{11}^* &= \left(\frac{1}{q_1} \int_0^T \left[p_i(q_1(t))x(t) - c_1(t)\right]dt\right)^{-1} \\
e_{12}^* &= \left(\frac{1}{q_2} \int_0^T \left[p_2 q_2(t)x(t) - c_2(t)\right]dt\right)^{-1} \\
e_{21}^* &= \left(\frac{1}{q_1} \int_0^T \left[p_i(q_1(t))x(t) - c_1(t)\right]dt\right)^{-1} \\
e_{22}^* &= \left(\frac{1}{q_2} \int_0^T \left[p_2 q_2(t)x(t) - c_2(t)\right]dt\right)^{-1}
\end{align*}
\]

NUMERICAL SIMULATIONS

CONCLUSION

In this talk, we devised a mathematical model which describes dynamics of a food chain composed by three different marine species; the aquatic plants, grazers and predators. We suggested two optimal harvesting policies. Based on the optimal control theory, we considered a first optimal harvesting policy for the environmental sustainability case, and which has aimed to minimize two fishing efforts related to grazer and predator states variables respectively for not affecting the trophic-halieutic environment, while finding the possibility to maximize their associated harvesting functions during fishing fleets, for the benefit of fishermen. As regards to the second optimal harvesting policy proposed for the bioeconomic case, it has aimed to minimize the two same fishing efforts functions but focusing only on the maximization of their associated profits functions, as an example of the effectiveness of the optimal harvesting strategy in the bioeconomy case.
The influence of price on the profit of fishermen exploiting prey, middle-predator and top-predator fish populations

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Introduction

An interesting and attractive topic in population ecology is understanding the diversity and community composition of populations that ultimately determine the overall stability of an ecosystem. In this work, we consider a tri-trophic prey-predator model which consists of three constituent populations; i.e., prey, middle predator and top predator. The objective of this work is to show how changes in the price can affect the profits of fishermen who exploit these three marine species. To achieve this aim we define a bioeconomic equilibrium model of these three populations then we compute the generalized Nash equilibrium point.

Problematic

**Bioeconomic model**

Let $x_1$, $x_2$ and $x_3$ respectively denoted the prey, middle predator and top predator populations sizes. The resulting system of equations reads as follows:

$$\frac{dx_1}{dt} = c_1 x_1 - \gamma_1 x_1 x_2 - \beta x_1 C_1,$$

$$\frac{dx_2}{dt} = c_2 x_2 - \gamma_2 x_2 x_3 - \beta x_2 C_2,$$

$$\frac{dx_3}{dt} = c_3 x_3 + \beta y_1 x_1 + \beta y_2 x_2 - C_3 x_3.$$

The problem of determining the fishing effort that maximizes the profit of each fisherman leads to a Nash equilibrium problem.

**Nash Equilibrium Problem (NEP)**

The first fisherman must solve the problem (P1): $\max_{x_1}(E) = c_1 x_1 - \gamma_1 x_1 x_2 - \beta x_1 C_1 \geq 0$, $E \geq 0$, $C_1$ given

The second fisherman must solve the problem (P2): $\max_{x_2}(E) = c_2 x_2 - \gamma_2 x_2 x_3 - \beta x_2 C_2 \geq 0$, $E \geq 0$, $C_2$ given

The essential conditions of Karush-Kuhn-Tucker applied to (P1): $2\gamma_1 x_1 - \gamma_1 x_1 x_2 - \beta C_1 = 0$, $c_1 - \gamma_1 x_1 - \beta C_1 = 0$

The essential conditions of Karush-Kuhn-Tucker applied to (P2): $2\gamma_2 x_2 - \gamma_2 x_2 x_3 - \beta C_2 = 0$, $c_2 - \gamma_2 x_2 - \beta C_2 = 0$

**Linear Complementarity Problem (LCP)**

Find vectors $u, w \in \mathbb{R}^3$ such that:

$$\begin{bmatrix} -2\gamma_1 x_1 - \gamma_1 x_1 x_2 - \beta C_1 & c_1 - \gamma_1 x_1 - \beta C_1 \end{bmatrix} \begin{bmatrix} u \ 0 \ 0 \end{bmatrix}, w \begin{bmatrix} 0 \ 0 \ c_2 - \gamma_2 x_2 - \beta C_2 \end{bmatrix} \geq 0.$$